

Generalized Stress Variables in Continuum Damage Mechanics

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Abstract

“Damage” in Continuum Mechanics is an internal variable used at the scale of a Representative Elementary Volume (REV) to model the decrease of stiffness due to the nucleation and propagation of crack tips in the bulk. Moreover, irreversible deformation can result from specific distributions of crack displacement vectors. This communication presents a work in progress on the thermodynamic consistency of Continuum Damage Mechanics (CDM) for quasi-brittle materials such as rock or concrete, in which “damage” actually lumps two effects: cohesive damage (i.e. propagation of crack tips) and adhesive damage (i.e. separation of crack faces). It is shown that state-of-the-art CDM models lumping cohesive and adhesive damage into the same internal variable are based on contradicting thermodynamic assumptions, and that the ambivalent definition of damage requires the use of generalized stress variables in a CDM framework. Corresponding work-conjugates of damage and crack-induced deformation are expressed for a general CDM model. Future work will investigate the conditions in which a hyper-elastic framework can be used in CDM.

Keywords: Continuum Damage Mechanics, thermodynamics, internal variable, irreversible deformation, cohesive damage, adhesive damage

1. Introduction

“Damage” in Continuum Mechanics refers to the loss of mechanical stiffness, defined at the scale of a Representative Elementary Volume (REV). Physically, damage is related to the nucleation and propagation of crack tips in the bulk of the REV [1]. Continuum Damage Mechanics (CDM) allows averaging the dissipation of energy due to the propagation of a distribution

of cracks. Micro-mechanical damage evolution laws derive from Linear Elastic Fracture Mechanics (LEFM) and homogenization schemes, and require assumptions on the system of interfaces, mainly: (1) the length probability density function, and (2) the space distribution (e.g., [2, 3, 4, 5]). In thermodynamic CDM models, postulates are made on the form of energy potentials instead. This second modeling option is convenient when material microstructure cannot be represented by a periodic pattern of defects or by a simple matrix-inclusion arrangement. This communication presents a work in progress on the thermodynamic consistency of CDM for quasi-brittle materials such as rock or concrete, in which “damage” actually lumps two effects: ‘cohesive damage’ (due to the propagation of micro-crack tips) and ‘adhesive damage’ (due to the separation of micro-crack faces). In the following analysis, the loss of cohesion is defined as bond breakage with no further separation of material surfaces, as opposed to the loss of adhesion, defined as the normal displacement of crack faces away from each other. According to these definitions, cohesive damage results in a degradation of elastic moduli, typically measured through loading and unloading cycles or acoustic emissions (e.g. [6, 7, 8] in rocks). After crack nucleation, both cohesive and adhesive damage can propagate and induce stiffness degradation and permanent deformation, which can be assessed through loading and unloading cycles and porosity and permeability measures (e.g., [9, 10, 11] in rocks). Models coupling damage (for stiffness evolution) and plasticity (for irreversible deformation) do not relate cohesive and adhesive damage to the state of microstructure, i.e. the two dissipative phenomena are coupled at the macroscopic level, but not at the microscopic level [12, 13, 14]. As a result, plastic deformation is not clearly related to the fabric [15, 16]. This is an important limitation of state-of-the art hydro-mechanical damage models of geomaterials [17, 18, 19, 20] - permeability models in particular [21, 11, 22, 23]. Section 2 explains how the existence of residual stresses induced by adhesive damage are usually accounted for in phenomenological CDM models. Section 3 discusses the nature of the stress (or driving force) variables that have to be employed in the damage potential in order to lump cohesive and adhesive damage into a single internal variable. Section 4 explains the advantages of using a truly hyper-elastic framework, and presents the main challenges encountered to date in doing so.

2. Residual Stress and Damage-Induced Irreversible Deformation

In CDM models of geomaterials, damage is most often defined as the second order crack density tensor [24] (the developments presented in the

following lead to the same conclusion if damage is a higher- or lower- order tensor). Cracks are viewed as slender inclusions. During damage propagation, crack length and aperture increase. Several formulas were proposed for the Helmholtz free energy of the damaged REV in order to account for crack-induced irreversible deformation, but in general, formulations can be recast into [25]:

$$\Psi_s(\boldsymbol{\epsilon}, \boldsymbol{\Omega}) = \frac{1}{2} \boldsymbol{\epsilon} : \mathbb{C}(\boldsymbol{\Omega}) : \boldsymbol{\epsilon} - \boldsymbol{\sigma}_R : \boldsymbol{\epsilon} \quad (1)$$

In which $\boldsymbol{\epsilon}$ and $\boldsymbol{\Omega}$ are deformation and damage respectively. $\mathbb{C}(\boldsymbol{\Omega})$ is the damaged stiffness tensor, and $\boldsymbol{\sigma}_R$ is the residual stress that needs to be supplied to the REV to close cracks, beyond a bare unloading [26]. Equation 1 is related to macroscopic observation: it is particularly convenient to use it when far-field deformation is controlled or measurable. Correspondingly, stress is assumed to be work-conjugate to total deformation, which yields:

$$\begin{aligned} \boldsymbol{\sigma} &= \frac{\partial \Psi_s(\boldsymbol{\epsilon}, \boldsymbol{\Omega})}{\partial \boldsymbol{\epsilon}} = \mathbb{C}(\boldsymbol{\Omega}) : \boldsymbol{\epsilon} - \boldsymbol{\sigma}_R \\ \mathbf{Y} &= -\frac{\partial \Psi_s(\boldsymbol{\epsilon}, \boldsymbol{\Omega})}{\partial \boldsymbol{\Omega}} = -\frac{1}{2} \boldsymbol{\epsilon} : \frac{\partial \mathbb{C}(\boldsymbol{\Omega})}{\partial \boldsymbol{\Omega}} : \boldsymbol{\epsilon} + \frac{\partial \boldsymbol{\sigma}_R}{\partial \boldsymbol{\Omega}} : \boldsymbol{\epsilon} \end{aligned} \quad (2)$$

Within the set of assumptions summarized in Eq. 1-2, the second law of thermodynamics writes:

$$\mathbf{Y} : \dot{\boldsymbol{\Omega}} \geq 0 \quad (3)$$

A sufficient condition to ensure thermodynamic consistency is to satisfy $\dot{\boldsymbol{\Omega}} \geq 0$ [27], which makes such a CDM framework very appealing. However, the underlying assumption when deriving the conjugation relationships in Eq. 2 is that $\boldsymbol{\epsilon}$ and $\boldsymbol{\Omega}$ are independent variables, which is somewhat in contradiction with the introduction of the residual stress $\boldsymbol{\sigma}_R$ to represent residual crack opening, and the resulting strain decomposition [28]:

$$\boldsymbol{\epsilon} = \boldsymbol{\epsilon}^{el} + \boldsymbol{\epsilon}^{ed} + \boldsymbol{\epsilon}^{id} = \boldsymbol{\epsilon}^E + \boldsymbol{\epsilon}^{id} \quad (4)$$

Noting \mathbb{C}_0 the undamaged stiffness tensor, $\boldsymbol{\epsilon}^{el} = \mathbb{C}_0^{-1} : \boldsymbol{\sigma}$ is the purely elastic deformation, which would occur in the absence of damage. $\boldsymbol{\epsilon}^{ed} = [\mathbb{C}(\boldsymbol{\Omega})^{-1} - \mathbb{C}_0^{-1}] : \boldsymbol{\sigma}$ is the additional elastic deformation induced by the loss of stiffness due to cohesive damage (i.e., crack tip propagation), assumed to be recoverable upon unloading [29, 30]. $\boldsymbol{\epsilon}^{id}$ is the residual deformation induced by adhesive damage (resulting from the separation of crack faces, and non-recoverable upon unloading). $\boldsymbol{\epsilon}^E = \boldsymbol{\epsilon}^{el} + \boldsymbol{\epsilon}^{ed}$ is the total elastic deformation (recoverable upon unloading).

3. Generalized Stress and Damage Driving Force

A decomposition of energy dissipation due to cohesive and adhesive damage should result in the following inequality for the second law of thermodynamics:

$$\boldsymbol{\sigma} : \dot{\boldsymbol{\epsilon}}^{id} + \mathbf{Y} : \dot{\boldsymbol{\Omega}} \geq 0 \quad (5)$$

A sufficient condition to ensure thermodynamic consistency (Eq. 5) is to define a positive and convex damage potential ϕ_d satisfying:

$$\dot{\boldsymbol{\epsilon}}^{id} = \frac{\partial \phi_d(\boldsymbol{\sigma}, \mathbf{Y})}{\partial \boldsymbol{\sigma}}, \quad \dot{\boldsymbol{\Omega}} = \frac{\partial \phi_d(\boldsymbol{\sigma}, \mathbf{Y})}{\partial \mathbf{Y}} \quad (6)$$

In order to enforce the normality rule with a single damage variable lumping the effects of cohesive and adhesive damage, it is necessary to define generalized “damage-driving” stress variables ($\tilde{\boldsymbol{\sigma}}$ or $\tilde{\mathbf{Y}}$) satisfying:

$$\dot{\boldsymbol{\epsilon}}^{id} = \frac{d\tilde{\phi}_d(\tilde{\boldsymbol{\sigma}})}{d\tilde{\boldsymbol{\sigma}}}, \quad \dot{\boldsymbol{\Omega}} = \frac{d\tilde{\phi}_d(\tilde{\mathbf{Y}})}{d\tilde{\mathbf{Y}}} \quad (7)$$

Let us examine the conditions of existence of such generalized stress variables. The inequality of dissipation can take the form in Eq. 5 only if stress is conjugate to the total elastic deformation $\boldsymbol{\epsilon}^E$:

$$\boldsymbol{\sigma} = \frac{\partial \Psi_s(\boldsymbol{\epsilon}, \boldsymbol{\Omega})}{\partial \boldsymbol{\epsilon}^E}, \quad \mathbf{Y} = -\frac{\partial \Psi_s(\boldsymbol{\epsilon}, \boldsymbol{\Omega})}{\partial \boldsymbol{\Omega}} \quad (8)$$

The Inequality of Clausius-Duhem writes:

$$\boldsymbol{\sigma} : \dot{\boldsymbol{\epsilon}}^E + \boldsymbol{\sigma} : \dot{\boldsymbol{\epsilon}}^{id} + \mathbf{Y} : \dot{\boldsymbol{\Omega}} - \frac{\partial \Psi_s}{\partial \boldsymbol{\epsilon}^E} : \dot{\boldsymbol{\epsilon}}^E \geq 0 \quad (9)$$

Due to the definition of stress in Eq. 8, a Legendre transform of Eq. 6 yields:

$$\dot{\boldsymbol{\epsilon}}^{id} = \frac{\partial \boldsymbol{\epsilon}^{id}}{\partial \boldsymbol{\epsilon}^E} : \dot{\boldsymbol{\epsilon}}^E + \frac{\partial \boldsymbol{\epsilon}^{id}}{\partial \boldsymbol{\Omega}} : \dot{\boldsymbol{\Omega}} \quad (10)$$

Formally, two choices are possible to determine the generalized stress variable conjugate to the lumped dissipation variable. Either the second law of thermodynamics is expressed in terms of damage:

$$\underbrace{\left(\boldsymbol{\sigma} + \boldsymbol{\sigma} : \frac{\partial \boldsymbol{\epsilon}^{id}}{\partial \boldsymbol{\epsilon}^E} \right)}_{\tilde{\boldsymbol{\sigma}}^E} : \dot{\boldsymbol{\epsilon}}^E + \underbrace{\left(\mathbf{Y} + \boldsymbol{\sigma} : \frac{\partial \boldsymbol{\epsilon}^{id}}{\partial \boldsymbol{\Omega}} \right)}_{\tilde{\mathbf{Y}}} : \dot{\boldsymbol{\Omega}} - \frac{\partial \Psi_s}{\partial \boldsymbol{\epsilon}^E} : \dot{\boldsymbol{\epsilon}}^E \geq 0 \quad (11)$$

Or in terms of irreversible deformation:

$$\underbrace{\left(\boldsymbol{\sigma} + \mathbf{Y} : \frac{\partial \boldsymbol{\Omega}}{\partial \boldsymbol{\epsilon}^E}\right) : \dot{\boldsymbol{\epsilon}}^E}_{\tilde{\boldsymbol{\sigma}}^E} + \underbrace{\left(\boldsymbol{\sigma} + \mathbf{Y} : \frac{\partial \boldsymbol{\Omega}}{\partial \boldsymbol{\epsilon}^{id}}\right) : \dot{\boldsymbol{\epsilon}}^{id}}_{\tilde{\boldsymbol{\sigma}}} - \frac{\partial \Psi_s}{\partial \boldsymbol{\epsilon}^E} : \dot{\boldsymbol{\epsilon}}^E \geq 0 \quad (12)$$

In order to use the generalized “damage-driving forces” introduced in Eq. 7, it is proposed to define a generalized elastic stress, conjugate to the total elastic deformation, by means of a free energy potential expressed in terms of total elastic deformation:

$$\tilde{\boldsymbol{\sigma}}^E = \frac{\partial \tilde{\Psi}_s(\boldsymbol{\epsilon}^E, \boldsymbol{\epsilon}^{id})}{\partial \boldsymbol{\epsilon}^E} = \frac{\partial \tilde{\Psi}_s(\boldsymbol{\epsilon}^E, \boldsymbol{\Omega})}{\partial \boldsymbol{\epsilon}^E} \quad (13)$$

Equality 13 constrains the choice of dissipation potentials introduced in Eq. 7. In order to close the formulation and develop the derivatives of the free energy (Eq. 1-2, 13), some postulates are needed to relate the damaged stiffness tensor $\mathbb{C}(\boldsymbol{\Omega})$, the residual stress $\boldsymbol{\sigma}_R$ and the irreversible deformation $\boldsymbol{\epsilon}^{id}$ to damage and total elastic deformation.

The damaged stiffness tensor can be obtained from the Principle of Equivalent Elastic Deformation (e.g., [1]) or from the Principle of Equivalent Elastic Energy (e.g., [31, 26]). Within the framework explained in Eq. 2 & 4, residual stress is related to damage and irreversible deformation as follows:

$$\boldsymbol{\sigma}_R = \mathbb{C}(\boldsymbol{\Omega}) : \boldsymbol{\epsilon}^{id} \quad (14)$$

Under the assumption of crack non-interaction, residual opening cannot be attributed to the interplay between stress concentrations induced by different families of cracks (characterized by their orientation). The emergence of irreversible crack-induced deformation has to be related to a microstructure perturbation other than the growth of inclusions - for instance: pore collapse and compaction around crack faces during crack opening, or gouge formation preventing crack closure. As a result, the assumption of crack non-interaction makes it possible to establish a one-to-one relationship between irreversible dilation in direction k and crack displacement vectors parallel to direction k . In state-of-the-art CDM models (e.g., [32, 33, 25]) $\boldsymbol{\sigma}_R$ is indeed assumed to be parallel to damage:

$$\boldsymbol{\sigma}_R = g\boldsymbol{\Omega} \quad (15)$$

In Eq. 14-15, it is assumed that residual stress and irreversible deformation only depend on damage - not on total elastic deformation. However, total

elastic deformation ϵ^E depends on damage by definition (Eq. 4). Therefore the partial derivatives of residual stress and irreversible deformation are expected to have a complex expression, unless ϵ^E and Ω are arbitrarily assumed to be independent variables, which leads to the same thermodynamic contradiction as the one highlighted in Section 2. The dependence of ϵ^E to damage brings complex expressions for the generalized stress variables ($\tilde{\mathbf{Y}}$ or $\tilde{\boldsymbol{\sigma}}$), which makes it challenging to express dissipation potentials ($\tilde{\phi}_d(\tilde{\mathbf{Y}})$ or $\tilde{\phi}_d(\tilde{\boldsymbol{\sigma}})$) according to physical processes. Resorting to a truly hyper-elastic framework [34] could overcome this limitation, since stress would be conjugate to the purely elastic deformation ϵ^{el} .

4. Pending questions in Hyper-Elasticity

In a hyper-elastic framework, the expression of the free energy depends on purely elastic deformation (ϵ^{el}) and damage (Ω), and stress writes:

$$\boldsymbol{\sigma} = \frac{\partial \psi_s(\epsilon^{el}, \Omega)}{\partial \epsilon^{el}}, \quad \mathbf{Y} = -\frac{\partial \psi_s(\epsilon^{el}, \Omega)}{\partial \Omega} \quad (16)$$

Note that hardening variables are not included in this discussion for simplicity. The strain decomposition in Eq. 4 establishes a dependency between the total elastic deformation and damage:

$$\epsilon^E = \mathbb{C}(\Omega)^{-1} : \mathbb{C}_0 : \epsilon^{el} \quad (17)$$

$$\dot{\epsilon}^E = \mathbb{C}(\Omega)^{-1} : \mathbb{C}_0 : \dot{\epsilon}^{el} + \underbrace{\left[\frac{d[\mathbb{C}(\Omega)^{-1}]}{d\Omega} : \mathbb{C}_0 : \epsilon^{el} \right]}_{\mathbf{A}(\epsilon^{el}, \Omega)} : \dot{\Omega} \quad (18)$$

If the inequality of dissipation is expressed in terms of damage, Eq. 18 yields:

$$\underbrace{\left(\boldsymbol{\sigma} : \mathbb{C}(\Omega)^{-1} : \mathbb{C}_0 + \boldsymbol{\sigma} : \frac{\partial \epsilon^{id}}{\partial \epsilon^{el}} \right)}_{\tilde{\boldsymbol{\sigma}}^{el}} : \dot{\epsilon}^{el} + \underbrace{\left(\mathbf{Y} + \boldsymbol{\sigma} : \frac{\partial \epsilon^{id}}{\partial \Omega} + \boldsymbol{\sigma} : \mathbf{A}(\epsilon^{el}, \Omega) \right)}_{\tilde{\mathbf{Y}}} : \dot{\Omega} - \frac{\partial \Psi_s(\epsilon^{el}, \Omega)}{\partial \epsilon^{el}} : \dot{\epsilon}^{el} \geq 0 \quad (19)$$

In terms of irreversible deformation:

$$\begin{aligned}
& \underbrace{\left(\boldsymbol{\sigma} : \mathbb{C}(\boldsymbol{\Omega})^{-1} : \mathbb{C}_0 + \boldsymbol{\sigma} : \mathbf{A}(\boldsymbol{\epsilon}^{el}, \boldsymbol{\Omega}) : \frac{\partial \boldsymbol{\Omega}}{\partial \boldsymbol{\epsilon}^{el}} + \mathbf{Y} : \frac{\partial \boldsymbol{\Omega}}{\partial \boldsymbol{\epsilon}^{el}} \right)}_{\tilde{\boldsymbol{\sigma}}^{el}} : \dot{\boldsymbol{\epsilon}}^{el} \\
& + \underbrace{\left(\boldsymbol{\sigma} + \boldsymbol{\sigma} : \mathbf{A}(\boldsymbol{\epsilon}^{el}, \boldsymbol{\Omega}) : \frac{\partial \boldsymbol{\Omega}}{\partial \boldsymbol{\epsilon}^{id}} + \mathbf{Y} : \frac{\partial \boldsymbol{\Omega}}{\partial \boldsymbol{\epsilon}^{id}} \right)}_{\tilde{\mathbf{Y}}} : \dot{\boldsymbol{\epsilon}}^{id} - \frac{\partial \Psi_s(\boldsymbol{\epsilon}^{el}, \boldsymbol{\epsilon}^{id})}{\partial \boldsymbol{\epsilon}^{el}} : \dot{\boldsymbol{\epsilon}}^{el} \geq 0
\end{aligned} \tag{20}$$

In order to close the formulation, some postulates are needed to relate the damaged stiffness tensor $\mathbb{C}(\boldsymbol{\Omega})$ and the irreversible deformation $\boldsymbol{\epsilon}^{id}$ to damage. Helmholtz free energy can be decomposed as:

$$\boldsymbol{\sigma} = \frac{1}{2} \boldsymbol{\epsilon}^{el} : \mathbb{C}_0 : \boldsymbol{\epsilon}^{el} + f(\boldsymbol{\epsilon}^{el}, \boldsymbol{\Omega}) \tag{21}$$

In which $\frac{1}{2} \boldsymbol{\epsilon}^{el} : \mathbb{C}_0 : \boldsymbol{\epsilon}^{el}$ is the purely elastic deformation energy stored in the undamaged counter-part of the solid skeleton, and $f(\boldsymbol{\epsilon}^{el}, \boldsymbol{\Omega})$ is a function of purely elastic deformation and damage, which represents the fabric of the REV, i.e. the shape and space distribution of defects. State-of-the-art CDM models resort to semi-empirical methods to define different damage tensors for different fabric effects [22]. Alternatively, micro-mechanical principles and homogenization schemes can be used to relate crack displacement vectors to a damage tensor $\boldsymbol{\Omega}$ (of order two or higher). For instance, for a distribution of penny-shaped cracks [4]:

$$f(\boldsymbol{\epsilon}^{el}, \boldsymbol{\Omega}) = \frac{1}{2} (\boldsymbol{\epsilon} - \boldsymbol{\epsilon}^{el}) : \mathbb{C}_d : (\boldsymbol{\epsilon} - \boldsymbol{\epsilon}^{el}) \tag{22}$$

In which tensor \mathbb{C}_d depends on the homogenization scheme chosen. Matrix-inclusion micro-mechanical models assume a decomposition by “species” (solid and void inclusions) whereas phenomenological frameworks are based on a decomposition by “phase” (elastic and non-elastic). In both theoretical frameworks, irreversible deformation induced by adhesive damage has to be accounted for by coupling the damage model to a plasticity model. This approach has the inconvenient to uncouple cohesive and adhesive damage at the microscopic level. It appears that the best option to ensure the thermodynamic consistency of hyper-elastic damage models for quasi-brittle materials is to develop a multi-scale modeling framework, in which rheological laws accounting for both geometric and non geometric effects are

coupled at the crack scale - not at the REV scale as it is currently done in CDM. Nevertheless, homogenized micro-mechanical models cannot capture non-geometrical effects of cracking, i.e. microstructure effects other than the initiation, propagation, growth and shrinkage of inclusions. Moreover, multiple forms of crack interactions raise challenging issues to define the REV size [35].

5. Conclusion

“Damage” in Continuum Mechanics refers to an internal variable used at the scale of a Representative Elementary Volume (REV) to model the decrease of stiffness due to the emergence of debonded surfaces in the bulk. Micro-mechanical models are based on matrix-inclusion homogenization schemes. Purely phenomenological models require postulates on the form of the functional used for energy potentials. These assumptions can be guided by the rheology expected with and without damage, or inferred from the expression of the damaged elasticity tensor obtained by homogenization - under certain micro-mechanical assumptions. In both micro-mechanical and phenomenological models, the free energy of the damaged material shall reflect the decrease of stiffness due to the decrease of effective material surface in the REV (cohesive damage), as well as the appearance of residual deformation due to the separation of crack faces (adhesive damage). Crack-induced irreversible deformation depends on the density and distribution of crack displacement vectors. In Continuum Damage Mechanics (CDM), damage variables are used to model both the propagation of crack tips (like in Linear Elastic Fracture Mechanics) and the separation of crack faces (with consequent irreversible deformation, like in plasticity). This modeling strategy avoids defining two dissipation potentials (one for damage of stiffness and one for plastic deformation). However, state-of-the-art CDM models lumping cohesive and adhesive damage into the same internal variable assume that total deformation and damage are two independent thermodynamic variables: this is in contradiction with the understated decomposition of deformation, which contains terms depending on damage. This communication shows that the ambivalence in the definition of damage requires the use of generalized stress variables in a CDM framework. The corresponding work-conjugates of damage and crack-induced deformation are expressed for a general CDM model. Although the methodology to close the formulation is provided, it is recognized that using one dissipation potential only adds complexity to express the lumped damage potential and the corresponding damage-driving forces, which become non-physical. It is anticipated that

a truly hyper-elastic framework could overcome the shortcomings of CDM models currently available to capture irreversible deformation induced by adhesive damage. Future work will be devoted to relate hyper-elastic CDM models to micro-mechanical principles and understand the impact of crack propagation on matrix perturbation (apart from the cracks). This research is expected to improve the assessment of multiple, interacting dissipation processes, and provide a methodology to determine the discontinuity scale at which most of the mechanical energy of a given thermodynamic system is dissipated.

References

- [1] J. Lemaître, R. Desmorat, Engineering Damage Mechanics. Ductile, creep, fatigue and brittle failure., Springer - Verlag, Berlin Heidelberg, 2005.
- [2] V. Deudé, L. Dormieux, D. Kondo, S. Maghous, Micromechanical Approach to Nonlinear Poroelasticity : Application to Cracked Rocks, Journal of Engineering Mechanics 128 (8) (2002) 848–855.
- [3] L. Dormieux, D. Kondo, F. Ulm, Microporomechanics, John Wiley & Sons, 2006.
- [4] Q. Zhu, D. Kondo, J.-F. Shao, Micromechanical analysis of coupling between anisotropic damage and friction in quasi brittle materials: Role of the homogenization scheme, International Journal of Solids and Structures 45 (2008) 1385–1405.
- [5] N. Xie, Q.-Z. Zhu, J.-F. Shao, L.-H. Xu, Micromechanical analysis of damage in saturated quasi brittle materials, International Journal of Solids and Structures 49 (6) (2012) 919–928.
- [6] A. Zang, C. F. Wagner, G. Dresen, Acoustic emission, microstructure, and damage model of dry and wet sandstone stressed to failure, Journal of Geophysical Research: Solid Earth (1978–2012) 101 (B8) (1996) 17507–17521.
- [7] A.-S. Chiarelli, J.-F. Shao, N. Hoteit, Modeling of elastoplastic damage behavior of a claystone, International Journal of Plasticity 19 (1) (2003) 23–45.

- [8] J. Fortin, S. Stanchits, S. Vinciguerra, Y. Guéguen, Influence of thermal and mechanical cracks on permeability and elastic wave velocities in a basalt from mt. etna volcano subjected to elevated pressure, *Tectonophysics* 503 (1) (2011) 60–74.
- [9] K. Chan, N. Brodsky, A. Fossum, S. Bodner, D. Munson, Damage-induced nonassociated inelastic flow in rock salt, *International journal of plasticity* 10 (6) (1994) 623–642.
- [10] M. Souley, F. Homand, S. Pepa, D. Hoxha, Damage-induced permeability changes in granite: a case example at the url in canada, *International Journal of Rock Mechanics and Mining Sciences* 38 (2) (2001) 297–310.
- [11] J. Shao, H. Zhou, K. Chau, Coupling between anisotropic damage and permeability variation in brittle rocks., *International Journal for Numerical and Analytical Methods in Geomechanics* 29 (12) (2005) 1231 – 1247.
- [12] J. Lubliner, J. Oliver, S. Oller, E. Onate, A plastic-damage model for concrete, *International Journal of Solids and Structures* 23 (3) (1989) 299–326.
- [13] N. Hansen, H. Schreyer, A thermodynamically consistent framework for theories of elastoplasticity coupled with damage, *Int. J. Solids and Structures* 31 (3) (1994) 359–389.
- [14] H. Egner, On the full coupling between thermo-plasticity and thermo-damage in thermodynamic modeling of dissipative materials, *International Journal of Solids and Structures* 49 (2012) 279–288.
- [15] M. Oda, Similarity rules of crack geometry in statistically homogeneous rock masses, *Mechanics of Materials* 3 (1984) 119–129.
- [16] M. Ortiz, A constitutive theory for the inelastic behaviour of concrete, *Mech. Mater.* 4 (1985) 67–93.
- [17] I. F. Collins, G. T. Houlsby, Application of thermomechanical principles to the modelling of geotechnical materials., *Proceedings: Mathematical, Physical and Engineering Sciences* 453 (1964) (1997) 1975–2001.
- [18] J. Shao, Poroelastic behaviour of brittle rock materials with anisotropic damage, *Mechanics of Materials* 30 (1) (1998) 41–53.

- [19] D. Gawin, F. Pesavento, B. Schrefler, Modelling of hygro-thermal behaviour of concrete at high temperature with thermo-chemical and mechanical material degradation, *Computer Methods in Applied Mechanics and Engineering* 192 (2003) 1731–1771.
- [20] A. P. Suvorov, A. P. S. Selvadurai, Effective medium methods and a computational approach for estimating geomaterial properties of porous materials with randomly oriented ellipsoidal pores, *Computers and Geotechnics* 38 (5) (2011) 721–730.
- [21] D. Kondo, L. Dormieux, Approche micro-mécanique du couplage perméabilité - endommagement, *Comptes-Rendus de Mécanique, Acad. Sci. Paris* 332 (2004) 135–140.
- [22] K. Maleki, A. Pouya, Numerical simulation of damage–Permeability relationship in brittle geomaterials, *Computers and Geotechnics* 37 (5) (2010) 619–628.
- [23] C. Arson, J.-M. Pereira, Influence of damage on pore size distribution and permeability of rocks, *International Journal for Numerical and Analytical Methods in Geomechanics* 37 (2013) 810–831.
- [24] M. Kachanov, Effective elastic properties of cracked solids: critical review of some basic concepts, *Appl. Mech. Rev.* 45 (8) (1992) 304–335.
- [25] C. Arson, B. Gatmiri, On damage modelling in unsaturated clay rocks, *Physics and Chemistry of the Earth* 33 (2008) S407–S415.
- [26] C. Arson, B. Gatmiri, Thermo-hydro-mechanical modeling of damage in unsaturated porous media: Theoretical framework and numerical study of the edz, *International Journal for Numerical and Analytical Methods in Geomechanics* 36 (2012) 272–306.
- [27] R. Desmorat, Positivité de la dissipation intrinsèque d’une classe de modèles d’endommagement anisotropes non standards, *Comptes Rendus Mécanique*.
- [28] R. K. Abu Al-Rub, V. George Z, On the coupling of anisotropic damage and plasticity models for ductile materials, *International Journal of Solids and Structures* 40 (2003) 2611–2643.
- [29] C. Arson, J.-M. Pereira, Rock stiffness and permeability during crack opening and closure: A planar transverse isotropic (pti) model using

- pore size distributions (psd), in: Proceedings of the 5th Biot Conference on Poromechanics, Vienna, Austria, 2013, pp. 2270–2279.
- [30] H. Xu, C. Arson, Anisotropic damage models for geomaterials: Theoretical and numerical challenges, *International Journal of Computational Methods*, Special Issue on Computational Geomechanics 11 (2) (2014) DOI: 10.1142/S0219876213420073.
 - [31] J. Cordebois, F. Sidoroff, Endommagement anisotrope en élasticité et plasticité, *Journal de Mécanique théorique et appliquée spec. issue* (1982) 45–60.
 - [32] D. Halm, A. Dragon, An anisotropic model of damage and frictional sliding for brittle materials, *Eur. J. Mech. A/ Solids* 17 (3) (1998) 439–460.
 - [33] G. Swoboda, Q. Yang, An energy-based damage model of geomaterials. I. Formulation and numerical results, *International Journal of Solids and Structures* 36 (12) (1999) 1719–1734.
 - [34] G. T. Houlsby, A. M. Puzrin, *Principles of hyperplasticity an approach to plasticity theory based on thermodynamic principles*, London, 2006.
 - [35] T. E. Lacy, D. L. McDowell, R. Talreja, Gradient concepts for evolution of damage, *Mechanics of Materials* 31 (12) (1999) 831–860.